

## NAG C Library Function Document

### nag\_prob\_durbin\_watson (g01epc)

#### 1 Purpose

nag\_prob\_durbin\_watson (g01epc) calculates upper and lower bounds for the significance of a Durbin–Watson statistic.

#### 2 Specification

```
void nag_prob_durbin_watson (Integer n, Integer ip, double d, double *pdl,
    double *pdu, NagError *fail)
```

#### 3 Description

Let  $r = (r_1, r_2, \dots, r_n)^T$  be the residuals from a linear regression of  $y$  on  $p$  independent variables, including the mean, where the  $y$  values  $y_1, y_2, \dots, y_n$  can be considered as a time series. The Durbin–Watson test (see Durbin and Watson (1950), Durbin and Watson (1951) and Durbin and Watson (1971)) can be used to test for serial correlation in the error term in the regression.

The Durbin–Watson test statistic is:

$$d = \frac{\sum_{i=1}^{n-1} (r_{i+1} - r_i)^2}{\sum_{i=1}^n r_i^2},$$

which can be written as

$$d = \frac{r^T A r}{r^T r},$$

where the  $n$  by  $n$  matrix  $A$  is given by

$$A = \begin{bmatrix} 1 & -1 & 0 & \dots & : \\ -1 & 2 & -1 & \dots & : \\ 0 & -1 & 2 & \dots & : \\ : & 0 & -1 & \dots & : \\ : & : & : & \dots & : \\ : & : & : & \dots & -1 \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

with the non-zero eigenvalues of the matrix  $A$  being  $\lambda_j = (1 - \cos(\pi j/n))$ , for  $j = 1, 2, \dots, n - 1$ .

Durbin and Watson show that the exact distribution of  $d$  depends on the eigenvalues of a matrix  $HA$ , where  $H$  is the hat matrix of independent variables, i.e., the matrix such that the vector of fitted values,  $\hat{y}$ , can be written as  $\hat{y} = Hy$ . However, bounds on the distribution can be obtained, the lower bound being

$$d_l = \frac{\sum_{i=1}^{n-p} \lambda_i u_i^2}{\sum_{i=1}^{n-p} u_i^2}$$

and the upper bound being

$$d_u = \frac{\sum_{i=1}^{n-p} \lambda_{i-1+p} u_i^2}{\sum_{i=1}^{n-p} u_i^2},$$

where  $u_i$  are independent standard Normal variables.

Two algorithms are used to compute the lower tail (significance level) probabilities,  $p_l$  and  $p_u$ , associated with  $d_l$  and  $d_u$ . If  $n \leq 60$  the procedure due to Pan (1964) is used, see Farebrother (1980), otherwise Imhof's method (Imhof (1961)) is used.

The bounds are for the usual test of positive correlation; if a test of negative correlation is required the value of  $d$  should be replaced by  $4 - d$ .

## 4 References

- Durbin J and Watson G S (1950) Testing for serial correlation in least-squares regression. I *Biometrika* **37** 409–428
- Durbin J and Watson G S (1951) Testing for serial correlation in least-squares regression. II *Biometrika* **38** 159–178
- Durbin J and Watson G S (1971) Testing for serial correlation in least-squares regression. III *Biometrika* **58** 1–19
- Farebrother R W (1980) Algorithm AS 153. Pan's procedure for the tail probabilities of the Durbin–Watson statistic *Appl. Statist.* **29** 224–227
- Imhof J P (1961) Computing the distribution of quadratic forms in Normal variables *Biometrika* **48** 419–426
- Newbold P (1988) *Statistics for Business and Economics* Prentice–Hall
- Pan Jie–Jian (1964) Distributions of the noncircular serial correlation coefficients *Shuxue Jinzhan* **7** 328–337

## 5 Parameters

- 1: **n** – Integer *Input*  
*On entry:* the number of observations used in calculating the Durbin–Watson statistic,  $n$ .  
*Constraint:* **n** > **ip**.
- 2: **ip** – Integer *Input*  
*On entry:* the number,  $p$ , of independent variables in the regression model, including the mean.  
*Constraint:* **ip**  $\geq$  1.
- 3: **d** – double *Input*  
*On entry:* the Durbin–Watson statistic,  $d$ .  
*Constraint:* **d**  $\geq$  0.0.
- 4: **pdl** – double \* *Output*  
*On exit:* lower bound for the significance of the Durbin–Watson statistic,  $p_l$ .
- 5: **pdu** – double \* *Output*  
*On exit:* upper bound for the significance of the Durbin–Watson statistic,  $p_u$ .
- 6: **fail** – NagError \* *Input/Output*  
The NAG error parameter (see the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_INT

On entry, **ip** =  $\langle value \rangle$ .  
Constraint: **ip**  $\geq$  1.

**NE\_INT\_2**

On entry, **n** =  $\langle value \rangle$ , **ip** =  $\langle value \rangle$ .  
 Constraint: **n** > **ip**.

**NE\_REAL**

On entry, **d** =  $\langle value \rangle$ .  
 Constraint: **d**  $\geq$  0.0.

**NE\_ALLOC\_FAIL**

Memory allocation failed.

**NE\_BAD\_PARAM**

On entry, parameter  $\langle value \rangle$  had an illegal value.

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

**7 Accuracy**

On successful exit at least 4 decimal places of accuracy are achieved.

**8 Further Comments**

If the exact probabilities are required, then the first  $n - p$  eigenvalues of  $HA$  can be computed and `nag_prob_lin_chi_sq (g01jdc)` used to compute the required probabilities with **c** set to 0.0 and **d** to the Durbin–Watson statistic.

**9 Example**

The values of  $n$ ,  $p$  and the Durbin–Watson statistic  $d$  are input and the bounds for the significance level calculated and printed.

**9.1 Program Text**

```

/* nag_prob_durbin_watson (g01epc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
    /* Scalars */
    double d, pdl, pdu;
    Integer exit_status, ip, n;
    NagError fail;

    INIT_FAIL(fail);
    exit_status = 0;
    Vprintf("g01epc Example Program Results\n");

    /* Skip heading in data file */

```

```
Vscanf("%*[^\\n] ");
Vscanf("%ld%ld%lf%*[^\\n] ", &n, &ip, &d);

g01epc(n, ip, d, &pdl, &pdu, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from g01epc.\\n%s\\n", fail.message);
    exit_status = 1;
    goto END;
}

Vprintf("\\n");
Vprintf(" Durbin-Watson statistic %10.4f\\n\\n", d);
Vprintf(" Probability for the lower bound = %10.4f\\n", pdl);
Vprintf(" Probability for the upper bound = %10.4f\\n", pdu);
END:
return exit_status;
}
```

## 9.2 Program Data

```
g01epc Example Program Data
10 2 0.9238
```

## 9.3 Program Results

```
g01epc Example Program Results
```

```
Durbin-Watson statistic      0.9238

Probability for the lower bound =      0.0610
Probability for the upper bound =      0.0060
```

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